**Basic Relativistic Mechanics**

Consider a stationary object of a certain mass, and all independent reference frames have coordinate axes parallel to and in the same direction as one another.

To an outside observer, let the mass of that object be m, and any event that occurs on the object is positioned at a point (x,y,z,t) in his reference frame. To someone else who is sitting on the object, the mass is measured as m', and the position of that same event is (x',y',z',t') in her own reference frame. Since neither observer is moving with respect to each other, then the values of m',x',y',z',t' equal exactly those of the outside observer.



Now if the object starts to move in the x (or x') direction and manages to attain a velocity v, the position of any event on the object can still be calculated in either reference frame. However, neither observer in his or her own reference frame will agree on the position or time of that event. If the person on the object, moving at velocity v, measures the position of the event as (x',y',z',t'), the outside observer will measure the position and time of that event as follows:



To the person travelling with the object, the mass of the object is m'. However, the outside observer will measure the mass of the moving object as:



Measuring momentum in different reference frames is again done in the same way: to the moving observer, the momentum of the object with respect to her is zero, and with respect to certain other objects in the universe, momentum is equal to p'=m'.v. However, the outside observer will measure the momentum of the moving object as:



At the same time, all other objects in the universe may (or may not) have zero momentum depending on which object the stationary observer is referring to in his calculations.

Suppose the person on the object, still moving with velocity v, decides to throw a ball forward with velocity w with respect to herself in the x' direction. To the outside observer, the velocity of the ball will be:



If the moving observer switched on the engines of the moving object, a force is exerted on the object (and eventually the observer). The magnitude of the force may be F'=m'.a to the moving observer, but the outside observer looking in will not accept this fact because he calculates the force on the object to be in fact:



In the throwing-the-ball example, suppose the ball had a mass of M' and moved with velocity w with respect to the moving observer. Its kinetic energy, KE, according to the moving observer, is:



To the outside observer, the kinetic energy of the ball with respect to him is:



The term on the left is called the total energy of the body, while M’c2 is called the body’s rest energy. Total energy is usually given the symbol E and can be related to momentum, p, of the body in the following way:

















Simplifying this to the form A2=B2+C2, we finally obtain:





From here, the famous E=M'c2 equation can be derived by setting momentum to zero in order to calculate the total energy of a non-moving object of mass mo.**Basic Vector Analysis**

Let us restrict our attention to vectors in three-dimensional space. No harm is done by doing so because the following results can be generalised to vectors in n-dimensions.

A vector is a quantity with magnitude and direction. Two vectors are said to be equal if, and only if, they have the same length and are both pointing in the same direction.

The components of a vector is usually denoted by **A**=a**i**+b**j**+c**k**. The letters **i**,**j**,**k** are unit vectors of magnitude 1 and direction corresponding to the X,Y,Z axes, respectively. Magnitude of a vector is defined as,



And the angles which the vector makes with the coordinate axes is easily obtained by solving the following equations:





Any vector can be made into a unit vector by dividing the component values by its magnitude;



Vectors may be added and subtracted.

**A**±**B** = (a**i**+b**j**+c**k**)±(d**i**+e**j**+f**k**)

= (a±d)**i**+(b±e)**j**+(c±f)**k**

Given two points in space, (a,b,c) and (d,e,f), a vector can be synthesised which passes through these two points. The vector is (d-a)**i**+(e-b)**j**+(f-c)**k**. Incidentally, the midpoint of the line separating these two points is given by:



The inner product (also called the dot product or scalar product) of two vectors is defined as:

**A.B** = **A****B** cos = ad+be+cf

This scalar product is zero only if the vectors **A** and **B** are both zero vectors, or if they are perpendicular to one another. One use for the scalar product is the evaluation of the angle (in radians) between two non-zero vectors:





The angle calculated here is defined as the smallest non-negative angle in the interval 0 when the vectors' tails are, in imagination, brought back to the origin. Two non-negative vectors are said to be 'parallel' if the angle between them is zero or , and 'perpendicular' if the angle between them is /2.

Another kind of multiplication called the vector (or cross) product, is defined as follows,



The magnitude **A**x**B**, which is also equal to **A****B**sin  where , is the angle between the vectors, represents the total area of the parallelogram determined by **A** and **B**. Also the vector **A**x**B** calculated from this productis in fact orthogonal to both **A** and **B**. The vector product of vectors is anti-commutative (that is, **A**x**B**=-(**B**x**A**)), and is not associative (that is, **A**x(**B**x**C**)(**A**x**B**)x**C**).



Instead of vectors having numerical components, we can have vectors whose components are functions of independent variables. In the most general form, a vector such as **A**(x,y,z) = f(x,y,z)**i**+g(x,y,z)**j**+h(x,y,z)**k** is called a *Vector Field*. For every point (a,b,c) in a vector field a unique vector starting at that point exists (that is, it will have its own special magnitude and direction at that point). The components f(x,y,z), g(x,y,z), and h(x,y,z) are called scalar fields.

The gradient of a scalar field f(x,y,z) in space is given by,



and it determines the rate of change of f(x,y,z) along the coordinate axes. But if one wanted to know the rate of change of f(x,y,z) along any vector in space, **A**=a**i**+b**j**+c**k**, we must apply the following:



Suppose we are given a vector field **A**(x,y,z)=f(x,y,z)**i**+g(x,y,z)**j**+h(x,y,z)**k**, and each scalar field f(x,y,z), g(x,y,z), and h(x,y,z) have partial first derivatives over some specified region, the 'Curl of **A**' is defined as:



The curl characterises the rotation in a field.

The Divergence of a vector field **A**(x,y,z) is defined by,



Applying the divergence to **A** results in a scalar function. When divergence is non-zero, it implies the presence of sources and sinks in a vector field.

When manipulating vector equations having divergence and curl operators as part of its structure, it may be possible to simplify the structure via the following vector formulas:

2f = .(f),

where 2 is called *The Laplace Operator*

xf = 0

(x**A**) = 0

x(x**A**) = (.**A**) - 2**A**

.(f**A**) = (.**A**)f + **A**.f

x(f**A**) = (x**A**)f + (f)x**A**

(**A**.**B**) = (**A**.)**B** + (**B**.)**A** + **A**x(x**B**) + **B**x(x**A**)

.(**A**x**B**) = **B**.(x**A**) - **A**.(x**B**)

x(**A**x**B**) = **A**(.**B**) - **B**(.**A**) + (**B**.)**A** - (**A**.)**B**